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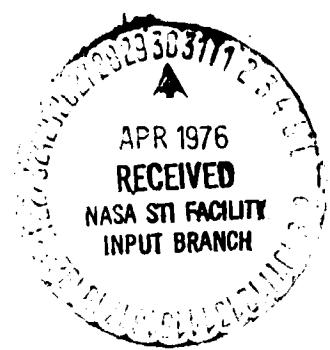
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**SIMULATION OF A TURBOFAN ENGINE FOR EVALUATION  
OF MULTIVARIABLE OPTIMAL CONTROL CONCEPTS**

by Kurt Seldner  
Lewis Research Center  
Cleveland, Ohio 44135

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## SIMULATION OF A TURBOFAN ENGINE FOR EVALUATION OF MULTIVARIABLE OPTIMAL CONTROL CONCEPTS

Kurt Seldner

Lewis Research Center

National Aeronautics and Space Administration  
Cleveland, Ohio

### ABSTRACT

The development of control systems for jet engines requires a real-time computer simulation. The simulation provides an effective tool for evaluating control concepts and problem areas prior to actual engine testing.

This paper describes the development and use of a real-time simulation of the Pratt & Whitney F100-PW-100 turbofan engine. The simulation is being used in a multi-variable optimal controls research program using linear quadratic regulator theory. The simulation is used to generate linear engine models at selected operating points and evaluate the control algorithm.

To reduce the complexity of the design, it is desirable to reduce the order of the linear model. This paper discusses a technique to reduce the order of the model and compares selected results between high and low order models.

The LQR control algorithms will be programmed on an SEL 810B digital computer. This computer will control the engine simulation over the desired flight envelope.

### INTRODUCTION

The development and evaluation of controls for aircraft propulsion systems requires a dynamic model of the engine and controls. A representative simulation provides the capability to predict engine performance over the specified flight envelope. The simulation also provides an effective tool to evaluate control concepts prior to hardware implementation on the actual engine. In addition, the simulation can serve as an aid in studying problems during the development and testing phases of the program.

This paper discusses methods of simulating aircraft propulsion systems with particular emphasis on a hybrid simulation of the Pratt & Whitney F100 afterburning turbofan engine. Lewis Research Center is involved in a joint NASA-Air Force controls program to evaluate a multivariable control for the F100 engine. The control will be based on linear quadratic regulator (LQR) theory. This theory requires that linear state variable models at various nominal operating points be known. This paper discusses a method to generate these models from the non-linear hybrid F100 simulation.

The F-100 engine model contains states that may not be measurable or the dynamics may not be significant for control design. Linear state regulator theory requires that all states are either measurable or observable. To reduce the complexity of the LQR multi-variable controller, the order of the linear models was reduced to include several selected states based on engine controls experience. A discussion of the methods employed to generate and reduce full state linear models is presented.

### COMPUTER MODELING OF ENGINES

Modern simulation techniques are mainly limited to digital and hybrid computers. The digital computer, due to its accuracy and repeatability, is generally preferred. However, real-time analysis cannot be easily accomplished with the digital computer. For this reason, the hybrid computer which incorporates the advantages of both analog and digital computers is ideally adapted to real-time simulation of aircraft engines.

A major problem encountered in modeling aircraft engines is to develop the bi-variate functions required to describe the performance of the rotating components of the engine. The digital computer can be programmed to generate these performance maps, whereas the analog computer is best utilized for calculations, such as integration, multiplication and single variable function generation.

Whenever combining digital and analog computers, for real-time simulations, one must consider the digital computer up-date time (time required to perform the map search and digital calculations) that cannot be exceeded if the closed loop accuracy and stability are to be maintained over the desired frequency range.

### ENGINE SIMULATION

To support the multi-variable optimal controls program, a real time hybrid simulation of the Pratt & Whitney F100 Series I afterburning turbofan engine was developed (Ref 1). Figure 1 is a schematic diagram, showing the various components of the engine. The approach used to simulate this type of engine is to model each component using basic conservation equations.

A digital simulation, developed by the engine manufacturer (P&W), was used to obtain both steady state and transient performance characteristics. The bivariate functions, defining the steady state performance of the compressor, fan and turbine were obtained from this simulation. These bivariate maps define the flow, pressure and enthalpy relationships as functions of fan and compressor rotational speeds.

The functions are stored in the digital part of the hybrid computer. The computer is capable of handling up to twelve functions of six pairs of independent variables. Whenever functions are highly non-linear (e.g. compressor map), special radial interpolation techniques are required. A digital subroutine has been developed for this type of function generation (Ref 2). Multi-variable functions of a single pair of independent variables can also be generated.

For modeling the F100 engine, volumes were assumed for components either where the gas dynamics are important or where algebraic loops must be avoided. The energy, momentum and continuity equations are solved for each volume. The rotor dynamics are computed from the conservation of angular momentum.

To satisfy the real time requirement, several model simplifications and assumptions were considered. To compute the temperature ratio across the compressor and fan, the efficiencies have to be included in the bivariate functions. To eliminate the additional map search, the temperature ratios were generated as piecewise linear functions of pressure ratio. Secondly, the gas properties were assumed constant, except for the torque balance relations, where the values for specific heat were scheduled as functions of compressor and fan discharge temperatures. This correction was necessary to improve the torque balance along the normal operating line.

#### MODEL VERIFICATION

To verify the engine model accuracy over the specified flight envelope, the required values for the control inputs must be supplied to the hybrid simulation. The control inputs for the F100 engine over the non-afterburning region are main burner fuel flow, exhaust nozzle area, inlet guide vane and stator vane position. The magnitudes of the control inputs and the engine output variables for various flight and operating conditions were obtained from the P&W digital simulation. This evaluation was performed to insure that the model was valid.

Figures 2-5 present the steady state hybrid results with the values obtained from the digital simulation. As can be observed, the comparison between the two simulations at the sea level static condition is satisfactory. Slight discrepancies can be observed for fan airflow at high altitude condition (figure 5). This error can be attributed to Reynolds number effects which were not included in the F100 Series I simulation. An updated simulation includes this effect.

The dynamic accuracy of the engine simulation must be acceptable over the specified flight envelope. The four control inputs, as previously defined for steady state operation, were scheduled as a function of time between idle and intermediate power conditions. The idle power condition is defined as the minimum engine thrust. The intermediate power condition is the maximum engine thrust before entering the engine afterburning region. Again, the scheduled control inputs were obtained from exercising the P&W simulation through transients. The effort was restricted to evaluating only the dynamic open-loop performance of the hybrid engine simulation by scheduling the control inputs instead of utilizing the P&W bill of materials controls.

Figure 6 illustrates the transient responses for fan speed and nozzle inlet pressure by advancing the power lever from idle to intermediate power condition. The results for the hybrid and digital models show that the hybrid simulation adequately represents the dynamics of the engine.

To verify the dynamic accuracy of the hybrid model, the transient responses of all significant output variables were determined over the complete flight envelope.

Because the engine simulation must also be valid for the afterburning region, both steady state and dynamic operation was evaluated at this condition. The additional control input, afterburner fuel flow, was scheduled as an input to the hybrid simulation. To study the transient performance, the power lever was advanced beyond the intermediate power condition to the full afterburning mode.

The approach used for recording and analyzing the transient data and, subsequently, the linear models and reduction of these linear models is shown by figure 7. The process of recording, reducing and plotting the data was completely automated. The link between the hybrid and IBM 360 computers is a SEL 810B digital computer, which was utilized to record the transient data and generate the information on paper tape. A 360 computer program was utilized to crossplot the hybrid transient data with the P&W transient data.

#### LINEAR MODELS

After the non-linear engine model evaluation was completed, the F100 hybrid simulation was utilized to generate linear models. The linear models are useful in analyzing the performance of the engine about an operating point and also for the linear quadratic regulator design.

Full state linear models were generated by adding a small disturbance to the nominal value of the state and control variables. The resultant deviations were recorded by the SEL 810B digital computer and then processed on the IBM 360 to compute the various system matrices (A, B, C, D). The procedure used to derive and reduce the state variable data is also illustrated by figure 7. Step responses for various control inputs can be generated directly on the IBM 360 computer.

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The results demonstrated by the hybrid and linear models for a main burner fuel flow step disturbance at the design condition are illustrated by figures 8-9. Some error can be observed for the final steady state values for both fan speed and exhaust nozzle pressure. These differences can be attributed to the non-linearities of the engine simulation. An analysis can be performed to determine the optimum values for state and control disturbances to restrict operation within the linear range. However, about an operating point extremely small disturbances will result in objectionable noise and recording errors in collecting the linear model data.

## MODEL REDUCTION

The primary objective of the F100 hybrid simulation is to test and evaluate the optimal multi-variable control algorithm. The linear quadratic regulator (LQR) design requires knowledge of the system matrices ( $A, B, C, D$ ) at various nominal operating points about the flight envelope. The linear models, described in the Model Verification Section, contain all states as defined by the hybrid simulation. Because LQR theory utilizes full state feedback (either measurable or estimated), the feedback control would be too impractical to develop and implement with the actual full state model engine representation. Therefore, it would be desirable to reduce the higher order linear model to an equivalent lower order model.

The difficulty encountered in the model reduction was the selection of appropriate states for the lower order model. The final choice was based strictly on past experience in selecting the significant states for this type of engine. The method applied to the F100 full state linear model was to partition the state vector into a set of slowly decaying states and a set of rapidly decaying states. The dynamics of the fast response states were neglected by setting the derivatives for these states to zero and approximating these states by their steady state values. The method is described in detail by reference 3.

The mathematical analysis is as follows:

Consider the linear system

$$\dot{x} = Ax + Bu \quad (1)$$

$$y = Cx + Du \quad (2)$$

where   
 $x$  = state vector  $n \times 1$   
 $u$  = control vector  $p \times 1$   
 $A$  = state matrix  $n \times n$   
 $B$  = control matrix  $n \times p$   
 $y$  = output vector  $r \times 1$   
 $C$  = output matrix  $r \times n$   
 $D$  = control matrix  $r \times p$

For the F100 model reduction, a sub-vector  $x_1$  consists of states that are to be included in the reduced model, whereas a sub-vector  $x_2$  includes the remaining states.

$$\text{Define } x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Equation (1) becomes

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} u \quad (3)$$

$$y = \begin{pmatrix} C_1 & C_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + Du \quad (4)$$

$A_{11}, A_{22}, B_1, B_2, C_1, C_2$  are sub-matrices resulting from the partitioning of the state vector  $x$

$x_1 = m \times 1$  sub-vector

$x_2 = (n-m) \times 1$  sub-vector

$A_{11} = m \times m$  matrix

$A_{12} = m \times (n-m)$  matrix

$A_{21} = (n-m) \times m$  matrix

$A_{22} = (n-m) \times (n-m)$  matrix

$B_1 = m \times p$  matrix

$B_2 = (n-m) \times p$  matrix

$C_1 = r \times m$  matrix

$C_2 = r \times (n-m)$  matrix

Assume that  $x_2 = 0$ . Solving eq. 3 for  $x_2$  and substituting for  $x_2$  into the  $\dot{x}_1$  equation yields

$$\dot{x}_r = \bar{A}x_r + \bar{B}u \quad (5)$$

$$y = \bar{C}x_r + \bar{D}u \quad (6)$$

where  $x_r$  is the reduced system state vector and the reduced system matrices are:

$$\bar{A} = (A_{11} - A_{12}A_{22}^{-1}A_{21}) \text{ (mmxm) matrix}$$

$$\bar{B} = (B_1 - A_{12}A_{22}^{-1}B_2) \text{ (mmp) matrix}$$

$$\bar{C} = (C_1 - C_2A_{22}^{-1}A_{21}) \text{ (rxm) matrix}$$

$$\bar{D} = (D - C_2A_{22}^{-1}B_2) \text{ (rpx) matrix}$$

Equations 5 and 6 represent the reduced order linear model. The system matrices ( $A, B, C, D$ ) can be replaced by the reduced system matrices ( $\bar{A}, \bar{B}, \bar{C}, \bar{D}$ ). These latter system matrices contain sufficient information to reconstruct the eliminated states and the output vector  $y$ .

The optimum reduction could be achieved by retaining only the slowly decaying states and eliminating the rapidly decaying states. If a reasonable separation exists between low frequency and high frequency states, then the lower order model will be a better approximation to the higher order model. The eigenvalues computed from the higher order model, derived from the hybrid simulation, are not widely spaced so that the reduced model is only approximate. In addition, a mode analysis indicates interaction between states. This implies that eigenvalues cannot be easily associated with their respective states. Depending on the sufficiency of the reduced model approximation, the eigenvalues will differ from those of the full state model.

Figures 10-12 compare selected output variable step responses for two reduced order with full state models. The fan speed was a retained state for the reduced order model, whereas the

high turbine inlet temperature was reconstructed. Thrust is an output quantity, which was reconstructed from the reduced order model. The results illustrate that the seventh order system is adequate and represents the full order model reasonably well. The difference between the various step responses can be attributed to the lack of high frequency information. This fact can be readily observed for both the fan and turbine inlet temperature where the response time is decreased as the model order is reduced. For the thrust response, due to the feed-forward term  $D_u$ , an initial thrust increase was observed for the third order reduced model. The effect is not as noticeable for the full state and seventh order models. Slightly slower response can be observed for the third order model. It should be emphasized that the selection of the retained states was based on intuition and engine control experience. Additional analysis is being performed to establish the final order of the reduced model and selection of states.

From these results it can be concluded that the full state model can be adequately represented by a third order model. The final decision for selecting the retained states depends upon the monitoring, protection and control problems.

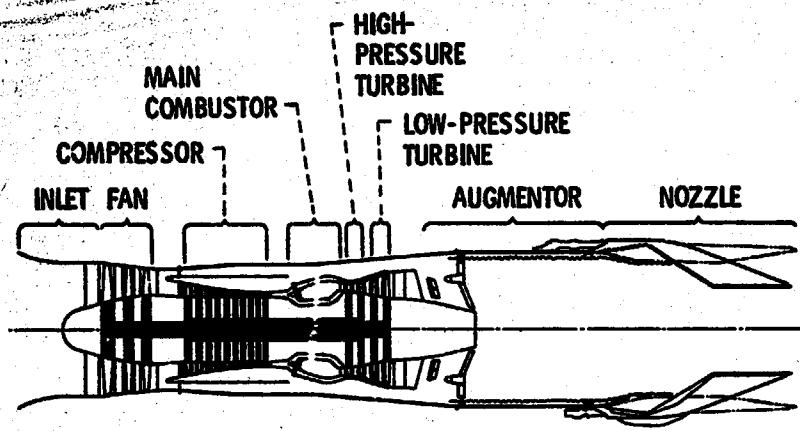
#### CONCLUSIONS

This paper has briefly discussed real-time simulation of an augmented turbofan engine. The results indicate that the hybrid computer is an effective tool to model engines efficiently and accurately. The steady state and transient performance of the model compares favorably with results obtained from the PW digital simulation.

The model can be utilized in evaluating modern control concepts and as a tool in solving problem areas during the development and testing phases of the program.

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- 2 Hart, C. E., "Function Generation Sub-programs for Use in Digital Simulations," TMX-71526, 1974, SA.
- 3 Blackburn, T. R., and Vaughan, D. R., "Application of Linear Optimal Control and Filtering Theory to the Saturn V Launch Vehicle," IEEE Transactions on Automatic Control, Vol. AC-16, No. 6, December 1971, pp. 799-806.



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Figure 1. - Schematic representation of F100-PW-100 augmented turbofan engine.

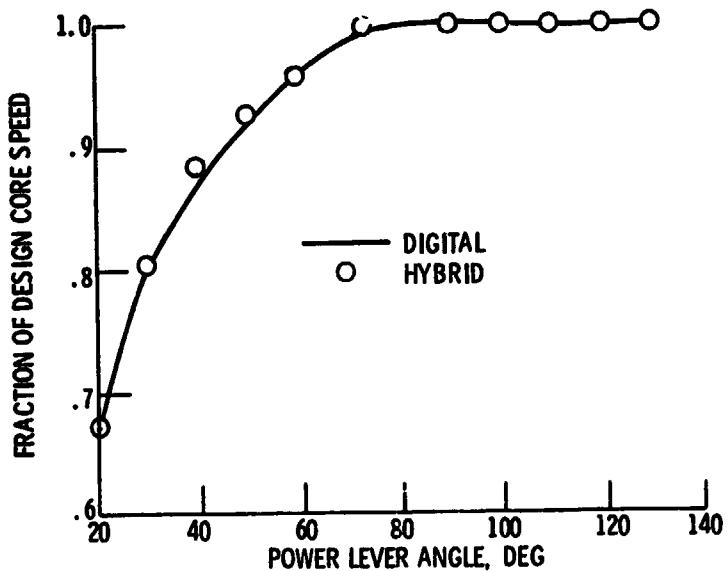


Figure 2. - Comparison of hybrid and digital steady-state data for compressor speed at sea level static.

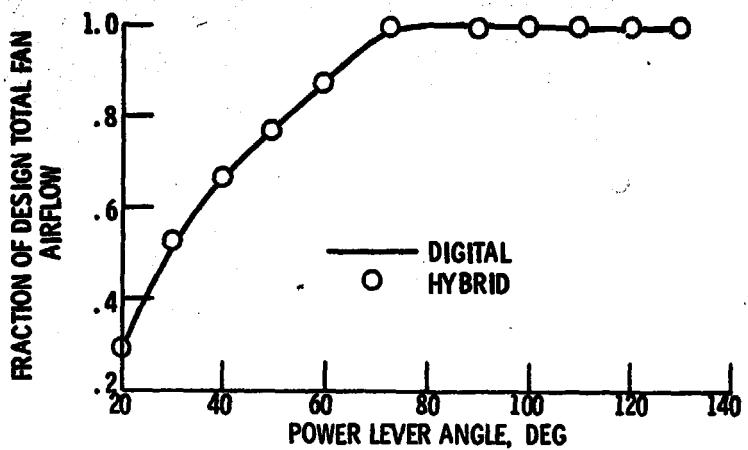


Figure 3. - Comparison of hybrid and digital steady-state data for fan airflow at sea level static.

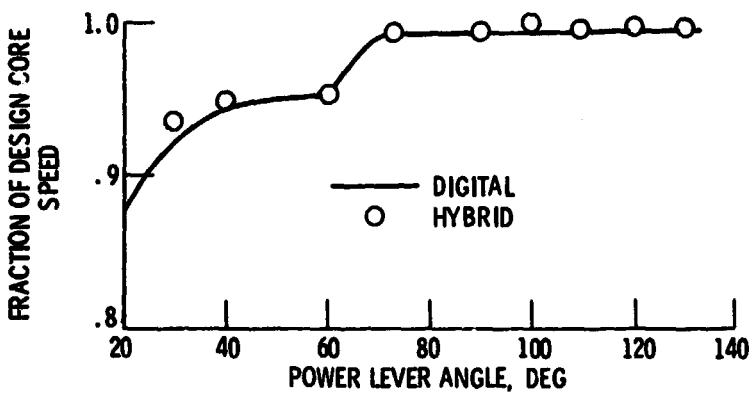


Figure 4. - Comparison of hybrid and digital steady-state data for compressor speed at altitude = 22 555 meters, Mach number = 2.5

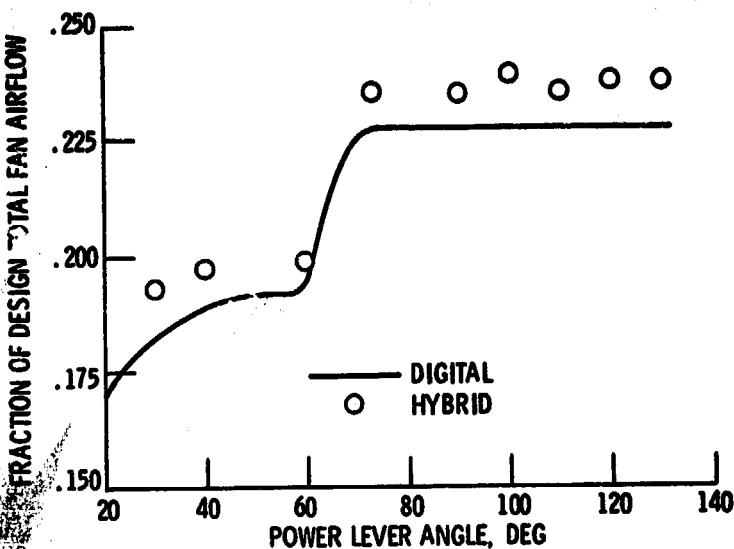


Figure 5. - Comparison of hybrid and digital steady-state data for fan airflow at altitude = 22 555 meters, Mach number = 2.5.

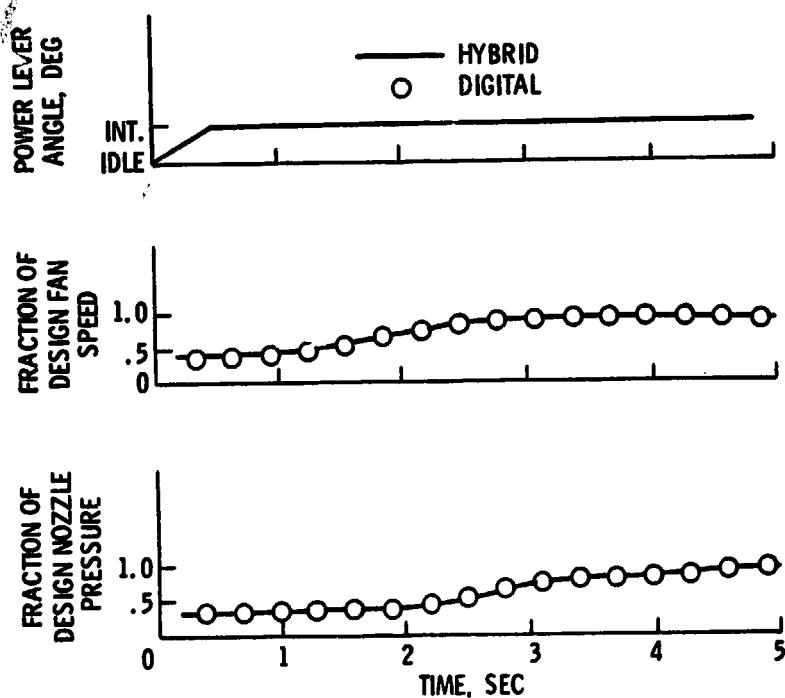


Figure 6. - Comparison of hybrid and digital simulation transient performance for idle-to-intermediate power lever advance, sea level static conditions.

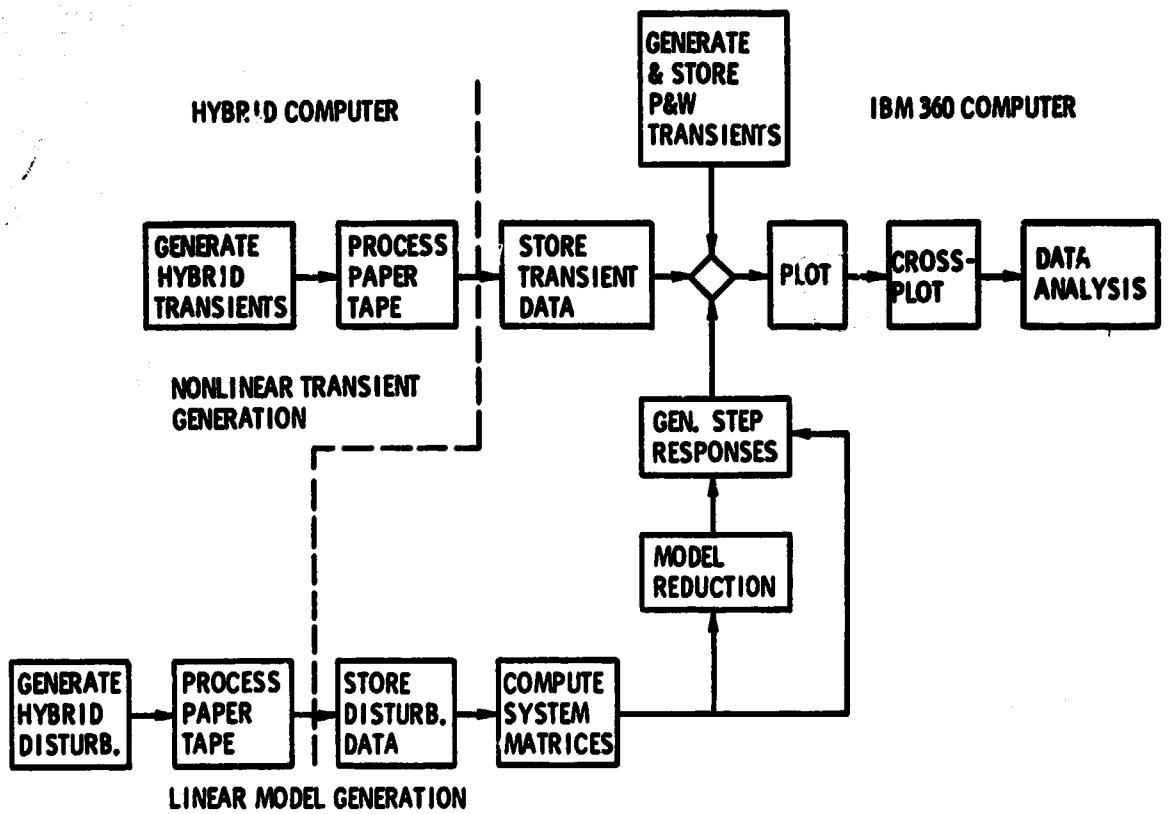


Figure 7. - Analysis of transient data and linear models.

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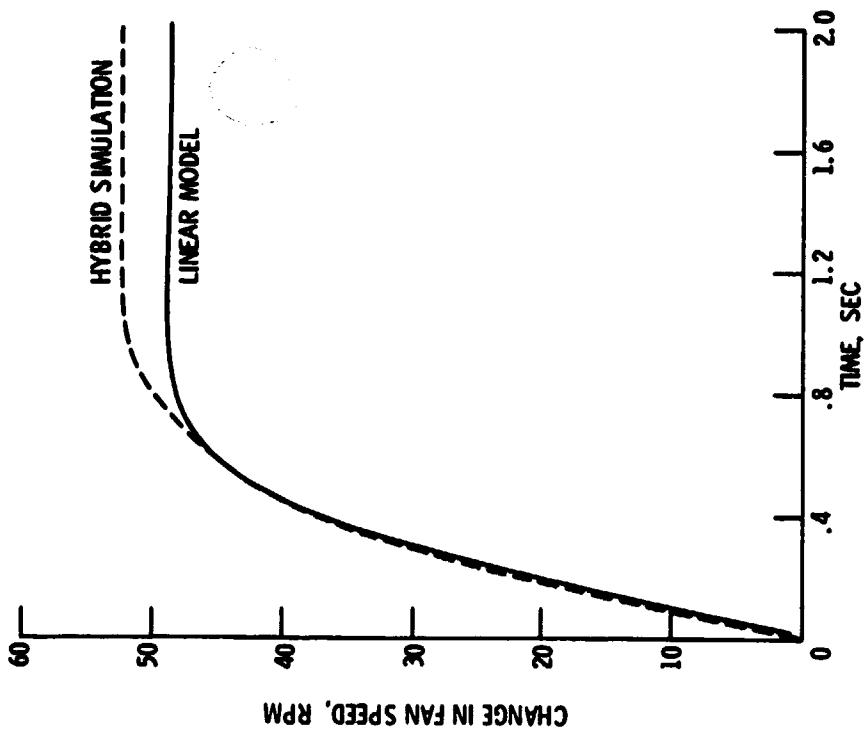


Figure 8. - Comparison between linear model and hybrid simulation for step change in fuel flow for sea level static condition.

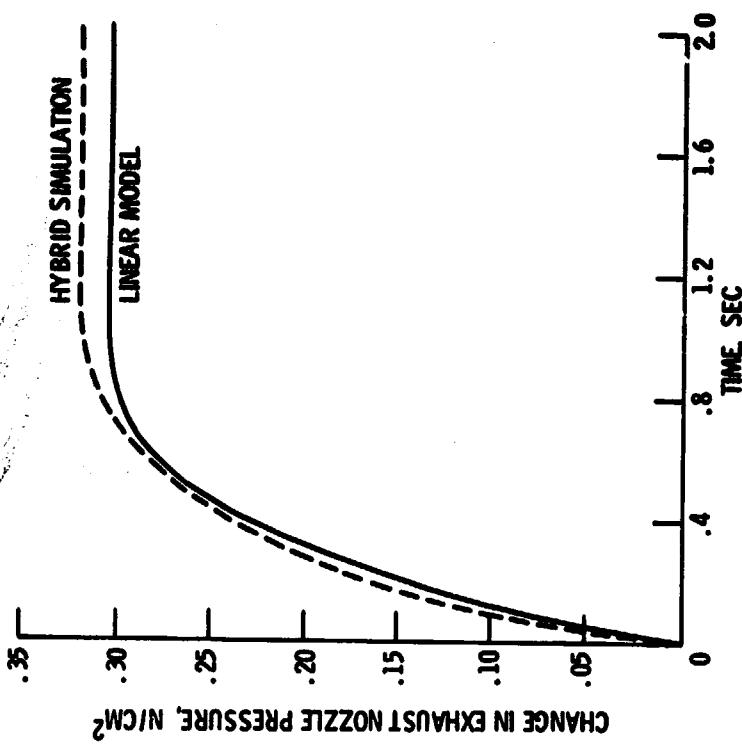


Figure 9. - Comparison between linear model and hybrid simulation for step change in fuel flow for sea level static condition.

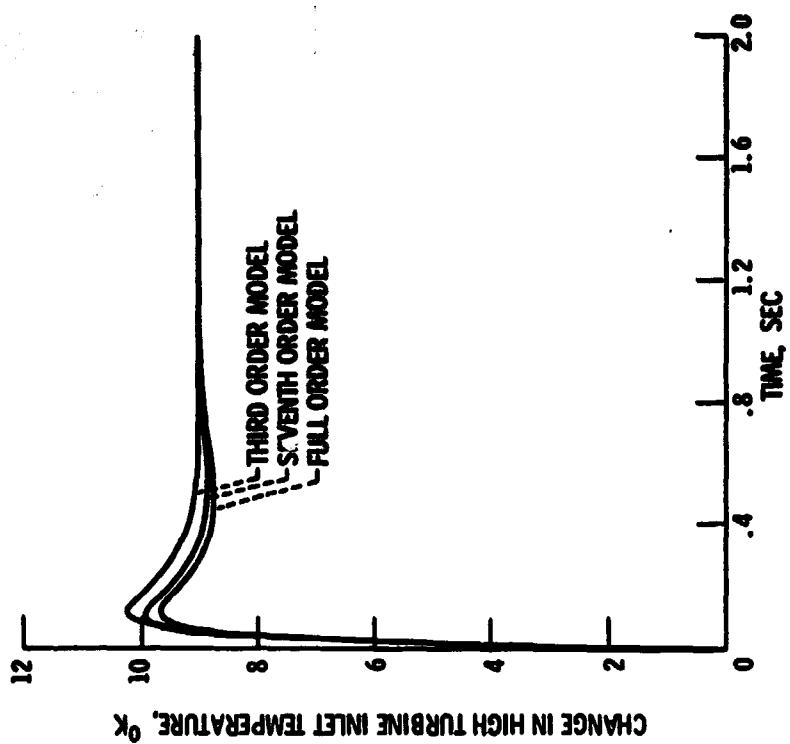


Figure 11. - Step responses for various linear models for fuel flow disturbance, sea level static condition.

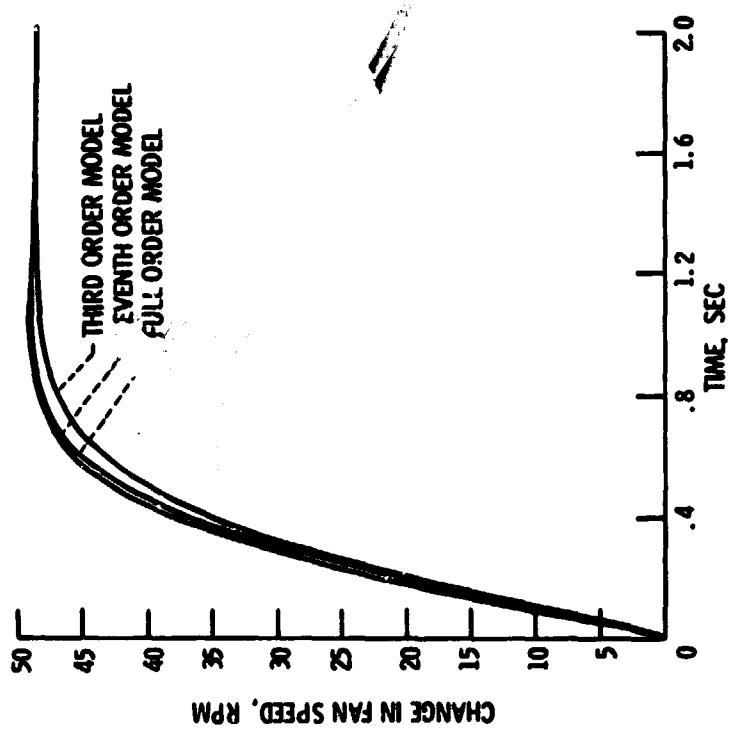


Figure 10. - Step responses for various linear models for fuel flow disturbance, sea level static condition.

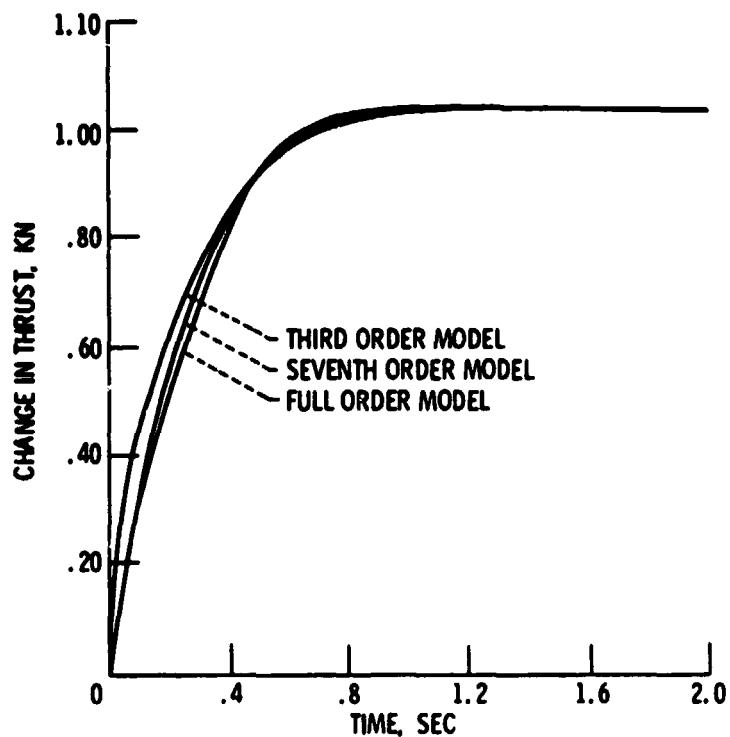


Figure 12. - Step responses for various linear models for fuel flow disturbance, sea level static condition.